A contribution to Post–Keynesian dynamic analysis

Contemporary Post–Keynesian macroeconomics focuses either on the determination of output in the short-period or on the determination of the growth rate of output in the long-period. By contrast, this paper presents a model which attempts to analyze *a sequence of short periods*, by investigating how each short-period position leads to the next one.

Domar (1947) focused on the dual effect of investment on the economy – it generates demand and output, and it increases productive capacity – and derived an *ideal* growth rate of investment that would make output grow exactly in line with capacity (the 'required growth path' or Harrod's 'warranted growth path'). By contrast, this paper uses a Kaleckian investment function to describe the *actual* path of investment and, taking into account its dual effect, analyzes the way it interacts with the *actual* paths of output and capacity.

The model improves our understanding of real world economies in two ways. First, it allows us to uncover some of the forces behind the several stages of the trade cycle – forces on the real side of the economy.[1] Secondly, the model helps us think dynamically about the impact of demand shocks (e.g. a fiscal stimulus).

[1] It should however be emphasized from the outset that a more complete explanation of the cycle should integrate other factors, in particular those analyzed by Minsky (1982) – expectations of entrepreneurs and their bankers, and the way these interact with financial conditions.

The model was inspired by Keynesian theories of the cycle, especially that of Harrod (1936, pp. 55-60 and pp. 89-101). There are however two differences. First and fundamentally, Harrod considered induced net investment to be equal to the *change in demand* times the capital-output ratio (the accelerator principle), whereas we consider it to be dependent upon the rate of utilization of productive capacity and thus upon the (relative) *level of demand*. As will be seen, this leads to a different depiction of the various phases of the trade cycle.

Secondly, Harrod's dynamic analysis is centred on the explanation of rates of change of variables, not of magnitudes of variables. By contrast, the analysis of this paper is dynamic in the sense that it is based on equations that provide recursive links between magnitudes of variables belonging to *different periods of time.*

Note finally that this paper's account of the cycle has connections not only with that of Harrod, but also with those of Kalecki (1933), Keynes (1936, chapter 22), Hicks (1950, chapter 8), Sherman (1991, 2010) and Harvey (2014). These connections will be pointed out along the text. Besides this, the dynamic analysis provided by this paper's model can – and will – be compared to the analysis of the transition between two long-run equilibrium positions of the economy in contemporary Kaleckian growth models (the so-called 'traverse'; see appendix I of this paper).

The paper is organized as follows. We begin with some stylized facts of the trade cycle. Afterwards, we present the equations of the model and use them to provide an account of the cycle. Finally, we use our model to analyze the effects of the U.S. fiscal policy of 2009-10.

Stylized facts

"As the [economic] system progresses in the upward direction, the forces propelling it upwards at first gather force and have a cumulative effect on one another, but *gradually lose strength* until at a certain point they tend to be replaced by forces operating in the opposite direction; which in turn gather force for a time and accentuate one another, until they too, having reached their maximum development, wane and give place to their opposite." Keynes (1936, pp. 313-4; italics added)

The main ideas of this paper were initially inspired by the cyclical behaviour of utilization, investment and profits – their marked rises in expansions and significant declines in recessions (Figures 1, 2 and 3).

<Figures 1, 2 and 3 around here>

In a second stage, the facts presented in Figure 4 and in Tables 1 and 2 led to an elaboration of those ideas. Figure 4 shows the behavior of *net* investment over U.S. cycles since 1947. As can be seen, net investment rose markedly over economic expansions and fell in recessions. However, net investment fell to negative values in only one recession; in the other recessions, it fell usually to zero and sometimes to positive values.

<Figure 4 around here>

Harvey (2014, pp. 396-7) calculated the annual percentage changes in GDP and in investment in three stages of the U.S. cycles observed since 1950: expansions except the last year; the last year of expansions; and recessions. The correspondent quarterly percentage changes in profits were also calculated.

An average of the figures for the cycles observed since 1950 is presented in Table 1. The major lesson that can be learnt is that expansions lose strength some time before they end. On average, in the last year of expansions GDP and investment growth slowed markedly, while the growth of profits stopped altogether.

<Table 1 around here>

Finally, Table 2 presents the average changes in capacity utilization in four stages of the U.S. cycles observed since 1967: the first two halves of expansions until utilization reached its peak; the last stage of expansions after the utilization peak; and recessions. Two conclusions can be drawn. First, the rise in utilization slows down significantly between the first and the second half of expansions before the utilization peak. Second, utilization starts to fall on average 9.6 months before the end of expansions (with one exception, between six months and one year before that end).

<Table 2 around here>

The model

The model assumes a closed economy with government. It is centred on two ideas. The first is the dual effect of investment on the economy. On the one hand, investment affects demand and output through the multiplier (which acts within a single period, i.e. without lags):

$$Y = \{1/[1- (c_{w.}(1- \pi) + c_p.\pi).(1- \tau)]\}. (I + C_p^* + G) \qquad (1)$$

Where Y is output, c_w and c_p are the marginal propensities to consume out of wages and out of profits, π is the profit share, τ is the overall tax rate, I is investment, C_p^* is the autonomous consumption of capitalists and G is government expenditure (for simplicity, time subscripts "t" are omitted).[2]

On the other hand, investment increases the production capacity of the economy. The effect of investment on production capacity is equal to net investment times the (potential) productivity of capital. Productive capacity is given by:

$$Y_{FC} = a.K_{-1} + a.(I - \partial.K_{-1}) \qquad (2)$$

Where a is the (potential) productivity of capital, assumed to be constant, K_{-1} is the capital stock and $\partial.K_{-1}$ is capital depreciation (both of the previous period). To make our main thesis clear, we will first assume that

[2] In an economy without government (hence $\tau = 0$) and with no saving out of wages (c_w=1), the multiplier would be reduced to the more familiar $1/(s_p.\pi)$, s_p denoting the marginal propensity to save out of profits.

net investment increases productive capacity *without lags.* Afterwards, we will explain that if net investment leads to the creation of capacity only after a lag – a more realistic assumption made by Kalecki (1933) - our argument is reinforced.

The second idea on which the model is based is that investment responds with a *lag* to deviations of the actual rate of utilization from a certain desired rate – an investment function akin to that used in contemporary Kaleckian growth models (cf. Lavoie, 2014, p. 361). Gross investment relative to the capital stock is given by:

$$I/K = \partial + I_A/K + \gamma.(\mathbf{u_{-1}} - u^*) \qquad (3)$$

Where I_A denotes autonomous investment, and u and u* represent the actual and the desired rates of utilization. The significance of autonomous investment will be explained later. Notice that *induced* investment responds only after a lag to economic conditions. This happens for several reasons (Sherman, 2010, p. 87). First, it takes time for businesses to know about changes in economic conditions. Second, businesses need time to *ponder* whether to advance with investments and to figure out the new facilities to build and the new machines and equipment to buy. Third, it may take time for businesses to obtain loans from banks or from bond issues. Finally, the construction of buildings requires government permits, which also takes time. As will be seen, it is the lagged effect of utilization on investment in our model that provides the link between each short-period position of the economy and the next one, and thereby allows us to trace out sequences of short-period positions.

The above Kaleckian investment function can be justified with two sorts of arguments. First, if the actual rate of utilization is above the desired rate, businesses will undertake positive net investment to increase their capital stock, and thereby try to reduce utilization towards the desired rate. In the opposite case, entrepreneurs will carry out negative net investment to reduce their capital stock and in that way try to raise utilization to the desired rate.

Second, because of fixed labour and capital costs changes in utilization over the cycle are associated with amplified changes in the profit rate and in total profits. This provides two further reasons for investment to be influenced by utilization. First, there is evidence that the actual profit rate influences the expected profit rate with a lag of three or four months (Klein and Moore, 1985, p. 254). Therefore, because it is linked to the actual profit rate, utilization is also related to the expected profit rate. Secondly, because it is associated with total profits, utilization is also linked to firms' financial capacity to invest – "an important part of investment is financed out of retained profits. Moreover, the amount that a company puts up of its own finance influences the amount it can borrow from outside" (Robinson, 1962, p. 86).

An account of the trade cycle

The engine that drives expansions: introduction

We start with a fundamental question of macroeconomics: *what makes aggregate demand grow* along expansions? According to mainstream economists, supply creates its own demand. The increases in aggregate demand along expansions are thus explained by the increases in aggregate supply in those periods. Post-Keynesian economists have a different view. They reject Say's Law, and argue that aggregate demand growth is instead determined by the growth of investment expenditure along expansions. While not proved, this view is suggested by the fact, presented in Table 1, that investment grows more than aggregate demand along expansions (and falls more than aggregate demand in recessions).

But the Post-Keynesian view poses another fundamental question: *what makes businesses increase investment year after year over expansions?* The answer implied by Keynes (1936, p. 313) is that investment depends on entrepreneurs' optimism, and this rises year after year along expansions. Instead, we propose an answer based on objective factors. This answer can be viewed as complementary to Keynes's and, in addition, may help explain how the growing optimism of entrepreneurs along expansions may come about.

We begin at the point of an expansion when utilization eventually rises above the desired rate. When this happens, entrepreneurs raise induced investment above the amount of capital depreciation (see equation 3 above) in an attempt to reduce utilization back towards the desired rate. If only a *single* individual entrepreneur acted in this way, his productive capacity would rise relative to his output, and therefore the rate of utilization would go down towards the desired level.

But when *many* entrepreneurs raise their investment above the amount of capital depreciation, besides increasing the productive capacity of the economy, they unconsciously provoke a macroeconomic effect: they increase aggregate demand and output. As a result, actual utilization does not necessarily fall back towards the desired rate. Instead, if the capacity effect (given by the productivity of capital, see equation 2 above) happens to be smaller than the aggregate demand effect (given by the multiplier, see equation 1 above), actual utilization will paradoxically move further above the desired rate.

Is the productivity of capital smaller than the multiplier? (i) Ponder first on the value of the multiplier. If we consider an overall tax rate of 0.4, the stylized facts c_p=0.4, c_w=0.9 and π= 0.4 mentioned by Lavoie (op. cit., p. 369 and p. 380) point to a multiplier in equation (1) of 1.72. Ninety percent of this value is associated with the initial change in investment expenditure plus the first and second rounds of consumption expenditure that follow it. Therefore, almost all of the effect of the multiplier occurs within a short period of time - probably one quarter, at most one semester. (ii) On the other hand, Lavoie (op. cit., p. 380) and Sherman (1991, p. 179) mention a productivity of capital of 1/3 per year (1/12 per quarter) as a stylized fact. (iii) Therefore, we can conclude that the productivity of capital, 1/12 per quarter, is smaller than the multiplier effect, 1.55 (= 0.9 * 1.72) exerted over one quarter.

The engine that drives expansions: a numerical example

Moving back to our core argument, we can now illustrate numerically how a self-sustained expansion may be brought about. Assume that the amount of capital depreciation is fixed at $100, that the productivity of capital per quarter is 1/10, and that the multiplier is 1.5 (the full operation of which requires one quarter; *if it required a longer period of time, the result would be the same*, as explained in appendix II).[3] In this setting, consider a period t of an expansion when utilization eventually rises above the desired rate. In response to this, in *period t+1* entrepreneurs will raise induced investment above the amount of capital depreciation, say from $100 to $110, in an attempt to drive utilization back to the desired rate. However,

[3] For simplicity, we neglect the fact that capital accumulation along the expansion will imply increasing amounts of capital depreciation, and assume this fixed at $100.

this will lead to a bigger increase in demand, \$10*1.5, than in productive capacity, \$10*(1/10), and therefore will end up in a paradoxical increase in utilization further above the desired rate. Output will rise according to demand and profits will rise in an amplified way. To fix ideas:

$\uparrow u_t$ above u*=> $\uparrow I_{t+1}$ above depreciation => $\uparrow demand_{t+1} > \uparrow capacity_{t+1}$ =>

=> $\uparrow u_{t+1}$ further above u*.

And this process – which may be called the ‘paradox of investment’ - will repeat itself over several periods. Indeed, the mentioned rise in utilization in t+1 will lead to a new increase in investment in t+2, which will again have a bigger effect on demand than on capacity, and thus will lead to a new rise in utilization in t+2. And so on.

Six notes on the argument just presented

First, along the way profits will rise with utilization and reinforce the upward movement. Second, the description so far assumes that entrepreneurs judge future rates of utilization and profit by their current levels. But if they develop a state of mind in which increasing utilization and profit rates lead them to expect further increases in the future, the boom will be exacerbated. Third, the above process may be the engine behind the sustained increases in utilization, profits and investment observed along expansions (Figures 1, 2 and 3).

Fourth, if we follow Kalecki (1933) and consider that investment orders do not lead to the creation of capacity instantaneously but only after a lag - say one year - the upward movement will be stronger. Indeed, in this case the amount of capacity created in a certain quarter of an expansion will not be determined by the amount of that quarter's net investment; instead, it will be determined by the *smaller* amount of net investment of the correspondent quarter of the previous year. As a result, the increase in capacity in each quarter of the expansion will be smaller, and thus – for the same increase in aggregate demand – the increase in utilization will be bigger. This being so, the upward movement will be stronger than if net investment leads to an instantaneous increase in capacity.

Fifth, besides providing an understanding of the self-sustained nature of expansions, the above argument has a more general application in macroeconomics – namely, it helps us move from a static to a dynamic analysis of demand shocks. Here is one example. The analysis of fiscal austerity is usually restricted to its multiplier effect on consumption and output in the short-period. But this short-period effect has an impact on the next short-period, and so on. Specifically, the initial decline in utilization in the short-period resulting from the multiplier effect of austerity leads to a reduction in investment and thus in utilization in the *next short-period,* and so on; that is to say, it depresses the paths of these two variables (and those of consumption and output) along a whole sequence of short periods. This being so, it is not surprising that the effects of the fiscal austerity implemented in the Euro Area after 2010 turned out to be much worse than initial forecasted (as recognized by the IMF (2012, pp. 41-3) itself). In fact, austerity affected not only consumption in the short-period, but also investment and consumption over a sequence of short periods. In particular, from 2010 to 2013 investment fell around 20 percent in Italy and Spain, 30 percent in Portugal and 45 percent in Greece (Ameco database).

Sixth, the above account of economic expansions can be compared with that of Harrod (1936, pp. 89-91). In Harrod, the increases in output along expansions are also brought about by rises in investment via the multiplier. And in turn those increases in output also lead to rises in investment through an investment function. But Harrod's investment function – the accelerator - is different, and implies that expansions can only occur along a path characterized by a dynamic equilibrium between demand and capacity and full rates of utilization. Moreover, that (warranted) path is unique and may only occur by coincidence.[4] By contrast, this paper's expansions are the result of a dynamic disequilibrium between demand and capacity and are associated with rising rates of

[4] Harrod only realized this in his subsequent work (Harrod, 1939). In this work, any divergence from the warranted growth path would lead either to collapse or to a path characterized by *increasing rates of growth* – an unrealistically explosive path.

utilization. They may thus occur along various paths and do not require any coincidence.

The expansion loses strength

In the real world utilization rates do not rise through the roof. In U.S. expansions, they have risen up to only 85-90 percent (Figure 1). So the question is: what does eventually tame the upward movement previously described?

Here is a possible answer. As investment grows period after period along an expansion, successive increases in investment continue to have a multiplier effect on demand and output of roughly the same size. But, *because they are associated with higher and higher levels of net investment (Figure 4)*, they generate larger and larger increases in the production capacity.[5] As a result, the increases in utilization – which depend positively on the increases in output but negatively on the increases in production capacity - become smaller and smaller.

Although progressively smaller, these rises in utilization still continue to lead to increases in investment – see equation (3) above - but at

[5] For example, the $10 increase in gross investment earlier in the expansion from $100 to $110 led to a net investment of $10 and to an increase in capacity of $10*(1/10); but the same $10 increase in gross investment later in the boom, say from $190 to $200, translates into a bigger net investment, $100, and into a bigger increase in capacity, $100*(1/10).

slower and slower rates. Therefore, the expansion loses strength. This argument is in line with the behavior of net investment, utilization, gross investment and output over U.S. expansions (Figure 4 and Tables 1 and 2).

The crisis

Once net investment has grown to a very high level, *a subsequent increase in investment will eventually start to have a smaller effect on demand than on capacity*. For example, at a very late stage of the expansion the same $10 increase in gross investment, say from $250 to $260, will still imply an increase in demand of $10*1.5 - but a bigger increase in capacity, $160*(1/10). Consequently, the paradox of investment ceases to hold: utilization falls. This in turn leads to a decline in investment and thus in output. In sum, once net investment has grown to a very high level:

$$\uparrow I_t \Rightarrow \uparrow demand_t < \uparrow capacity_t \Rightarrow \downarrow u_t \Rightarrow \downarrow I_{t+1} \Rightarrow \downarrow output_{t+1}$$

Note that this argument is different from Harrod's account of crises: according to this, investment falls because – as a result of a tendency of the propensity to consume to decline along expansions - consumption eventually ceases to rise rapidly enough to justify the net investment being made. (See Harrod, 1936, pp. 90-1 and pp. 94-5). The above argument is also distinct from the old 'over-investment' account of crises. According to this, investment falls because at the end of the boom there is too much capital – in other words, the rate of capacity utilization is *too low*.[6] This view is not supported by the data (Figure 1). In contrast, according to our argument investment falls because at the end of the boom a further rise in investment leads to a *decline* in utilization, a view that is generally consistent with the data (Figure 1 and Table 2).[7]

On the other hand, it should be said that the decline in utilization at the end of the boom is just one of the factors contributing to the observed crises. In fact, and first, some observed economic crises – including the most acute ones – may be the mere result of financial crises, whose origins and contours have been explained by Minsky (1982). Secondly, the declines in investment that lead to some crises may also be caused by the reductions in profits triggered by the rising costs in raw materials that typically occur in the last third of expansions (Figure 5; see also Sherman, 1991, p. 222 and p. 259); with one or two exceptions, real interest rates do not rise significantly in the second half of expansions and thus cannot

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[6] In Keynes's words (1936, pp. 320-1): "[According to the over-investment theory, at the end of the boom] every kind of capital-goods is so abundant that no new investment is expected, even in conditions of full-employment, to earn in the course of its life more than its replacement cost."

[7] Through a different route from ours, Kalecki (1933) also arrives at the conclusion that the decline in utilization is the proximate cause of crises: "after an interval of time has elapsed" since the increases in investment orders of an expansion, the actual delivery of investment goods starts exceeding "the level of replacement requirements, [and thus] the volume of capital equipment starts to increase. Initially this restrains the rate at which investment activity is increasing, and at a later stage causes a decline in investment orders." (p. 9)

contribute to explain those declines in investment (Figure 6). Last but not least, some crises may be primarily caused by the too optimistic expectations that entrepreneurs tend to develop over a boom:

"It is an essential characteristic of the boom that investments which will in fact yield, say, 2 per cent in conditions of full-employment are made in the expectation of a yield of, say, 6 per cent … When the disillusion comes, this expectation is replaced by a contrary 'error of pessimism', with the result that the investments, which would in fact yield 2 per cent in conditions of full-employment, are expected to yield less than nothing; and the resulting collapse of new investment then leads to a state of unemployment in which [those] investments … in fact yield less than nothing." (Keynes, 1936, pp. 321-2).

<Figures 5 and 6 around here>

The recession

As mentioned, according to our model the decline in utilization at the end of the boom leads to a reduction in investment and thus in output. One consequence is a decline in profits. On the other hand, net investment falls to a level that is at first positive (Figure 4). Therefore, capacity keeps on rising, at a time when output is declining.[8] As a result, there is *a new*

[8] If we consider that net investment affects capacity only after a lag, the continuation of the increase in capacity does not result from net investment falling to a level that is first

decline in utilization which, along with the dwindling profits, produces a further decline in investment. And so on. Needless to say, if entrepreneurs develop a state of mind in which declining utilization and profit rates lead them to expect further declines in the future, the recession will be intensified. This account of recessions is in line with the data (Figures 1, 2 and 3).

Note finally that, because of his view that net investment is determined by the accelerator, Harrod (1936) presents a more drastic account of the recession. According to him, as soon as an initial decline in investment reduces output via the multiplier, additional capital goods are not required and thus net investment is rapidly reduced to zero or negative values. "Thus the conjoint action of the [Accelerator] and the Multiplier accounts for the catastrophic nature of the slump" (pp. 97-8).

The revival

Before World War 2, some revivals of economic activity might have occurred for the following reason (Kalecki, 1933, p. 11; Harrod, 1936, p. 58; Keynes, 1936, p. 318). Once gross investment dropped to very low levels, it could not fall much further and would become stagnant – and, as a result, the same would happen with demand and output. A period would

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positive. Instead, it results from the fact that the high levels of net investment of the last quarters of the expansion are now having their lagged effect on capacity.

then arise when investment remained below the amount of capital depreciation - implying an erosion of capacity - and output was more or less stagnant. This state of affairs would lead to a gradual increase in utilization, which would eventually induce a revival of investment.

By contrast, in post-war recessions the U.S. capital stock has barely decreased – with one exception, gross investment has at most fallen to the level of capital depreciation (Figure 4). Therefore, the pre-war explanation of revivals no longer applies. Which factors may then explain the revivals of investment that have initiated post-war economic expansions? The answer is probably the following one.

A significant part of investment is not related to the rate of capacity utilization. Instead, it includes investment associated with innovations, housing investment associated with population growth, and investment which is only expected to pay for itself over a long period and is thus linked to the expected long–run growth of sales (e. g. a hydroelectric dam). This autonomous part of investment is related to the overall size of the economy and is thus subject to a *rising trend*. This being so, the pre-war explanation of revivals may be recast in the following way (Hicks, 1950, p. 105).

Once *induced* gross investment drops to very low levels in a recession, it cannot fall much further. Therefore, its decline begins to be offset by the rising trend of autonomous investment and thus to be associated with a recovery of overall investment. In turn, this leads to an increase in utilization (Figure 1) and thereby to a new economic expansion.[9]

Finally, it should be noted that two other aspects may have also contributed to the revivals of investment that initiated post-war economic expansions: (i) the positive effect on profits of the sharp decline in the cost of raw materials relative to consumer prices that typically occurred in the post-1970 recessions (Figure 5; see also Sherman, 1991, p. 222 and p. 261); (ii) and, less importantly, the declines in real interest rates that occurred in two or three recessions (Figure 6).

The effects of the expansionary U.S. fiscal policy of 2009-10

We now illustrate how the model can help us think dynamically about the impact of demand shocks, by using it to analyze the effects of the U.S. expansionary fiscal policy of 2009-10.

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[9] Because Hicks' induced investment depends on the accelerator rather than on the rate of utilization, he formulates this last part of the revival in a different way. According to him, the resumption of overall investment makes output start to rise and this, by the accelerator, resuscitates induced investment. And this resurrection leads to a new upswing based on the interaction between the accelerator and the multiplier along the lines of Harrod.

The effect on the path of output

The stimulus package of the Obama Administration of 2009-10 translated into a rise in total U.S. government spending between the first and the third quarters of 2009, followed by stabilization at a high level until the third quarter of 2010. Afterwards, the expiration of the stimulus package led to a decline in total government spending. All this is shown in Figure 7.

<Figure 7 around here>

According to short-period multiplier analysis, this behavior of government spending should have led to rises in output from mid-2009 to mid-2010, followed by declines in output afterwards. Thus, the U.S. Congressional Budget Office estimated the effect of the Obama stimulus on GDP shown in Figure 8. And, based on this, Krugman (2011) argued that **"**the U.S. federal government has been practicing destructive fiscal austerity since the middle of 2010 - and that's not even talking about what's happening at the state and local level."

<Figures 8 and 9 around here>

Yet, instead of falling output kept on rising after 2010 (Figure 9). How was this possible in view of the ´destructive austerity'? The model presented in this paper suggests the following answer. The Obama stimulus led to a revival of economic activity after the middle of 2009. This in turn led to rises in utilization and profits, which afterward produced a revival of business investment in the beginning of 2010 (Figures 10, 11 and 12). As a result, a dynamic interplay between rising utilization and profits and increasing investment followed – and this brought about a continuous expansion of output. (Without the reversal of the increase in government spending, the same would have happened but at a faster pace).

<Figures 10, 11 and 12 around here>

The effect of expansionary policy on the debt-to-GDP ratio

We now analyze the implications of our argument on the debate about the effects of fiscal policy on the debt-to-GDP ratio, the indicator most used to assess the sustainability of public finances.[10]

It is possible to argue that expansionary policy tends to reduce the debt-to-GDP ratio in the *short-term* (Leão, 2013). (i) Through the multiplier an increase in government spending raises output – the denominator of the ratio. (ii) On the other hand, the higher GDP brings about larger tax revenues and lower government social transfers. Therefore, the rise in government spending translates only partially into an increase in

[10] It should be said that, according to Modern Monetary Theory (MMT), the sustainability of public finances does not depend on the path of the debt-to-GDP ratio. Even so, the discussion that follows is important for two reasons. First, MMT's claim may not be valid in an open economy under fixed exchange rates. Secondly, many economists are unaware or reject MMT's claim.

debt – the numerator of the mentioned ratio. (iii) Since it raises both the numerator and the denominator, a rise in government expenditure has *a priori* an uncertain effect on the debt-to-GDP ratio. (iv) However, if we do the arithmetic using estimates of the relevant parameters (the multiplier, the tax rate and the impact of a higher output on social transfers), we conclude that a rise in government spending raises public debt by a smaller percentage than GDP – and therefore leads to a lower debt-to-GDP ratio.

However, according to the theory of the multiplier this is only a short-term result. The reason is that when the fiscal stimulus is withdrawn output falls back to its initial level – *but the larger debt remains*. Thus, after a brief decline, the debt-to-GDP ratio rises above its level before the stimulus.[11]

By contrast, according to this paper's model (i) if the stimulus is withdrawn only after it has started a virtuous spiral of rising utilization, profits and private investment, output will grow continuously. (ii) In turn, the growing output will generate swelling tax revenues and decreasing government social transfers – and thus lead to a continuous improvement of the budget balance and of the path of public debt. (iii) Finally, the

[11] The result will be the same if the stimulus is *not* withdrawn and government spending stays constant at the higher level. In fact, while in this case output will stay constant rather than fall back to its initial level, the budget deficit will remain. Therefore, public debt will keep on growing period after period, and so will the debt-to-GDP ratio.

decelerating (or declining) debt and the growing GDP will lead to a continuous deceleration (or reduction) of the debt-to-GDP ratio.

The evolution of the U.S. public finances after 2010 illustrates this point. The economic expansion that followed the Obama stimulus led to a big decline in the budget deficit, from almost 10 percent in 2009 to a little over 2 percent of GDP in 2015 (Figure 13). This combination of dwindling budget deficits and rising nominal GDP led in turn to a halt in the increase of the debt-to-GDP ratio after 2012 (Figure 14).

<Figures 13 and 14 around here>

The contrast with the evolution of the Eurozone is worth noting (Table 3). After the beginning of austerity in 2011, Eurozone's output remained roughly stagnant for several years: GDP in 2015 was only 1.3 percent higher than in 2011 (the U.S. GDP was 8.8 percent higher). And because the stagnant output prevented an automatic growth in tax revenues and an automatic reduction in government social transfers, the budget deficit declined by only 2.2 percent of GDP between 2011 and 2015 - despite all the austerity. Over the same period the U.S. budget deficit fell by 6.1 percent of GDP.

<Table 3 around here>

Summary and conclusions

Contemporary Post-Keynesian macroeconomics focuses on the analysis of the short-period or on the study of the long-period. In contrast, this paper analyzed how successive short-period positions lead to one another. By marrying Domar's dual effect of investment with a Kaleckian investment function, the paper argued that successive short-period positions are linked through the following mechanism:

Utilization and profits of a period t influence investment of *period t+1*. This in turn affects utilization and profits of period t+1, which afterwards influence investment of period t+2. And so on.

Based on this dynamic interaction, the paper presented the following account of the trade cycle. (i) The growth of aggregate demand along expansions is driven by the following mechanism. When in a period t utilization eventually rises above the desired rate, in period t+1 entrepreneurs respond by raising induced investment above the level of capital depreciation, in an attempt to drive utilization back to the desired rate. However, this leads to a bigger increase in demand than in capacity, and therefore ends up in a paradoxical increase in utilization in t+1. Output rises according to demand and profits rise in an amplified way.

And this process – the paradox of investment - repeats itself over several periods. In fact, the mentioned rise in utilization in t+1 leads to a new increase in investment in t+2, which has again a bigger effect on demand than on capacity, and thus leads to a new rise in utilization in t+2. And so on. Along the way profits rise markedly with utilization and reinforce the upward movement.

(ii) As investment grows period after period along an expansion, the successive increases in investment continue to have roughly the same multiplier effect on demand. But, because they are associated with increasingly higher levels of net investment, they generate larger and larger increases in production capacity. As a result, the paradox of investment loses strength: the increases in utilization become smaller and smaller. Although progressively smaller, these rises in utilization still continue to lead to increases in investment – but at slower and slower rates. As a result, the expansion loses strength.

(iii) Once net investment has grown over the expansion to a very high level, a further rise in investment will start to have a smaller effect on demand than on capacity. Consequently, the paradox of investment ceases to hold: utilization falls. This in turn leads to a decline in investment and in output: a crisis.

(iv) The reduction of output then causes a decline in profits and, coupled with a capacity that continues to rise, generates a *new* decline in utilization. In turn, these declines produce a further reduction in investment and thus in output. And so on.

(v) Once induced gross investment drops to very low levels, it cannot fall much further. Therefore, its decline begins to be offset by a rising trend of autonomous investment and thus to be associated with a recovery of overall investment. In turn, this leads to an increase in utilization and thereby to a new economic expansion.

Two final notes about the account of the cycle just presented. First, it suggests that a cycle does not constitute the path of an economy returning to a static equilibrium after being disturbed by an exogenous shock. Instead, it suggests that cycles are an inevitable feature of capitalist economies resulting from an *intrinsic instability of private investment*. Second, the above account does not constitute a complete explanation of the cycle. In particular, and as mentioned along the text, the explanation of the upper and lower turning points of the cycle should include other important factors.

Using fiscal policy as an example, the last pages of the paper illustrated how the model can help us think dynamically about the impact of demand shocks. According to static multiplier analysis, a fiscal stimulus raises GDP and lowers the debt-to-GDP ratio in the short-term. However, when the fiscal stimulus is withdrawn, output falls back to its initial level and the debt-to-GDP ratio rises above its level before the stimulus.

By contrast, according to the dynamic analysis developed in this paper, if the fiscal stimulus is carried out for a period long enough to start a virtuous spiral of rising utilization, profits and investment, it will set off an economic expansion. In turn, this will lead to a continuous reduction of the budget deficit and to a correspondent slowdown in the growth of public debt. And this, combined with the growing GDP, will lead to a deceleration and eventual reduction of the debt-to-GDP ratio.

Appendix I

This appendix makes a brief comparison between this paper's dynamic analysis and the analysis of the traverse in contemporary Kaleckian growth models (see Lavoie, 2014, pp. 361-6).

Like in our dynamic analysis, the transition between two long-run equilibrium positions in Kaleckian growth models – following for example a change in the propensity to save – also involves an interplay between changes in utilization and changes in investment (or rather in the rate of accumulation). There is however a difference: in Kaleckian growth models accumulation affects utilization *only* through its effect on demand and output, whereas in this paper's model investment affects utilization also through its effect on production capacity.[12]

[12] In the Kaleckian growth model, the influence of accumulation on utilization is given by the following equation: $g = s_p.\pi.u.a$, where g is the rate of accumulation, s_p is the marginal propensity to save out of profits, π is the profit share, u is the rate of utilization and a is the (potential) productivity of capital (cf. Lavaoie, op. cit., pp. 361-2).

Now, this equation is deduced from the Keynesian equation in which output is determined by aggregate demand without its combination with an equation indicating *the effect of net investment on production capacity*. Indeed, denote output by Y, aggregate demand by AD, consumption by C, investment by I, total wage income by W, total profit income by P, capital by K and the profit rate by r. Thus, in a closed economy without government and no saving out of wages: $Y = AD \Leftrightarrow Y = C + I \Leftrightarrow W + P = [W + (1-s_p).P] + I \Leftrightarrow s_p.P = I \Leftrightarrow s_p.P/K = I/K \Leftrightarrow s_p.r = g \Leftrightarrow r = g/s_p$ (the Cambridge Equation). Given Weisskopf's decomposition of the profit rate, $r = \pi.u.a$, the Cambridge Equation becomes $\pi.u.a = g/s_p \Leftrightarrow g = s_p.\pi.u.a$. We can thus conclude that in the Kaleckian growth model accumulation affects utilization through its effect on demand and output, but not through its effect on production capacity.

In addition, this difference leads the two models to arrive at significantly different conclusions. In Kaleckian growth models, the transitions end up in new long-run positions characterized by constant rates of accumulation and constant rates of capacity utilization. Moreover, these constant rates of utilization are in general different from the desired rate - a conclusion that some authors find difficult to accept (e.g. Skott, 2012, pp. 117-25).[13]

In turn, our dynamic analysis generates perpetual oscillations of investment and utilization over the cycle. Rates of utilization are thus also in general different from the desired rate. However, there is a reason for that: even though *individual* entrepreneurs are always trying to achieve their desired rate of utilization, because of the paradox of investment actual utilization always ends up going above or below that desired rate.

[13] For a Kaleckian reaction to this difficulty, see Lavoie (op. cit., pp. 388-90 and pp. 402-6).

Appendix II

Our explanation of the self-sustained nature of expansions in the main text (pp. 10-11) assumed that the operation of the multiplier requires only one quarter. The objective of this appendix is to explain that, if it instead requires more than one quarter, that explanation will still hold. The reason is that the relation between the increases in investment in the various quarters of an expansion and the contemporaneous increases in demand will still end up being given by the full value of the multiplier.

Suppose first that the operation of the multiplier requires *one semester* (say, because the first and the second rounds of consumption expenditure only take place in the quarter *after* the increase in investment that generates them). In this case, the $10 increase in investment that takes place in the *first* quarter of the expansion described in the main text leads to an increase in aggregate demand of only $10 in that first quarter (instead of $15). However, in the second quarter of the expansion demand will rise by $15: $10 as a result of the increase in investment in that second quarter plus $5 associated with the increases in consumption resulting from the increase in investment in the previous quarter. Extending this reasoning forward leads to the conclusion that the $10 increases in investment in the various quarters of the expansion end up being associated with $15 increases in demand in the corresponding quarters: $10 of increases in investment plus

$5 of increases in consumption resulting from the increases in investment in the preceding quarters.

A quick look at Table 4 leads to the conclusion that the same happens if the operation of the multiplier requires *three quarters* (say, because the first and the second rounds of consumption expenditure take place, respectively, in the first and the second quarters *after* the quarter of the increase in investment that generates them). The only difference is that it now takes three instead of two quarters for the $10 increases in investment to start generating $15 increases in aggregate demand.

<Table 4 around here>

General conclusion: if the multiplier requires more than one quarter to exert its effect, the relation between the increases in investment in the various quarters of an expansion and the contemporaneous increases in demand still ends up being given by the full value of the multiplier; the only difference is that it will take more than one quarter for that to start happening. This being so, the explanation of the self-sustained nature of expansions presented in the main text remains valid if the multiplier requires more than one quarter to exert its effect.

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Figure 1. Capacity utilization over U.S. cycles, 1967-2017

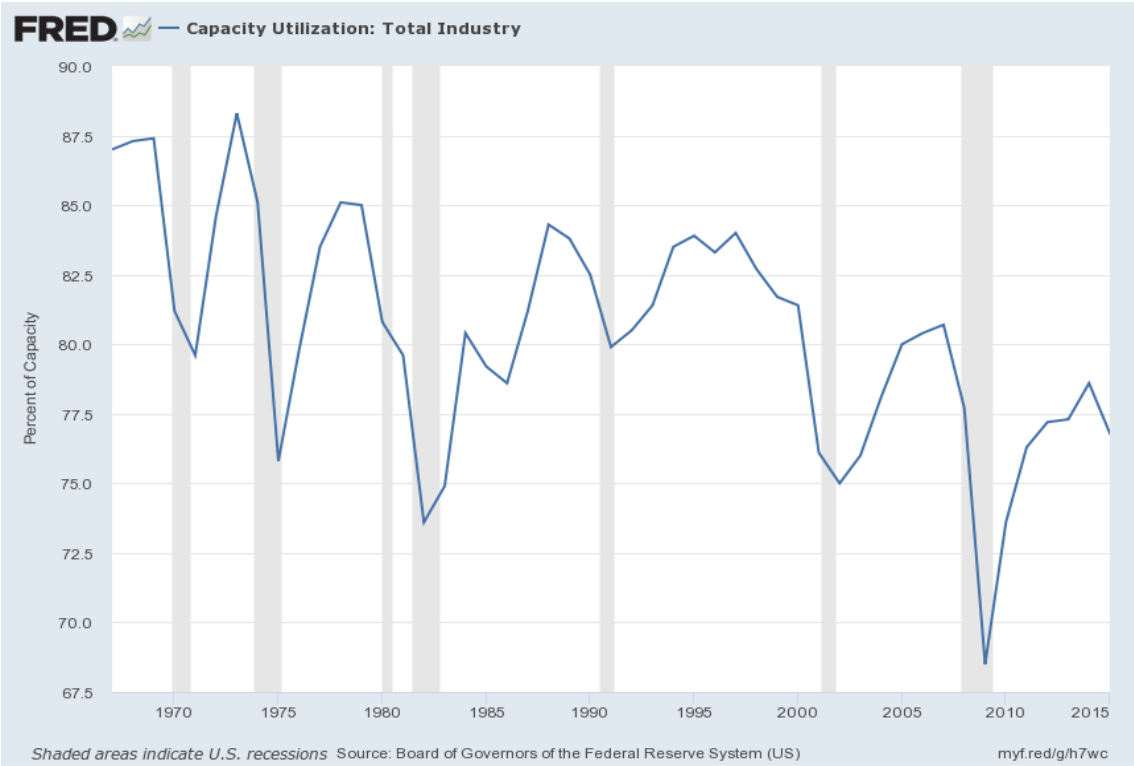


Figure 2. Share of private investment in GDP over U.S. cycles, 1947-2017

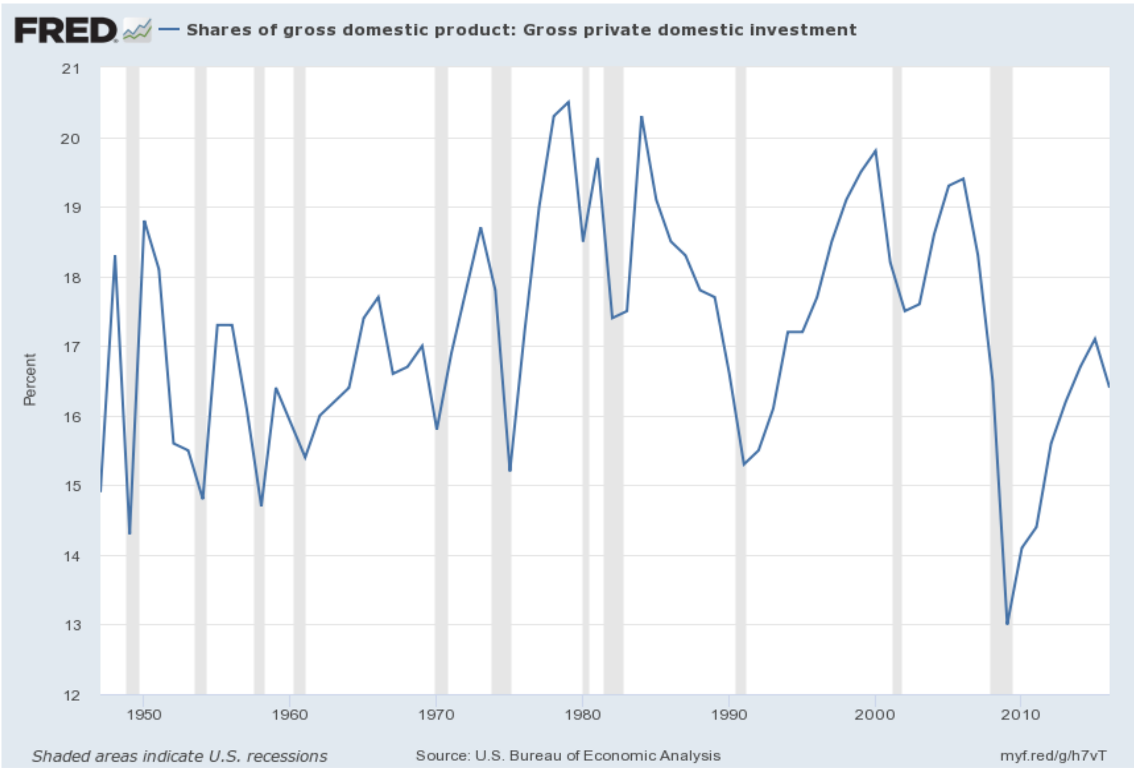


Figure 3. Log of profits over U.S. cycles, 1947-2017

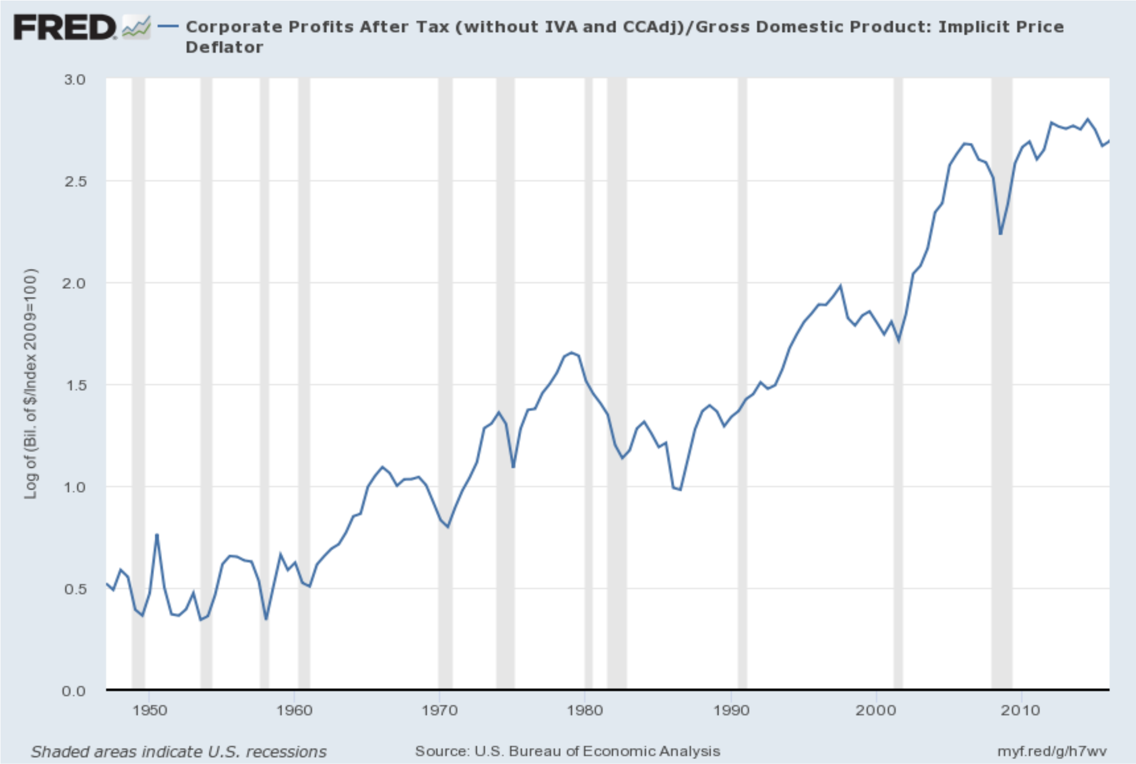


Figure 4. Net investment over U.S. cycles, 1947-2017

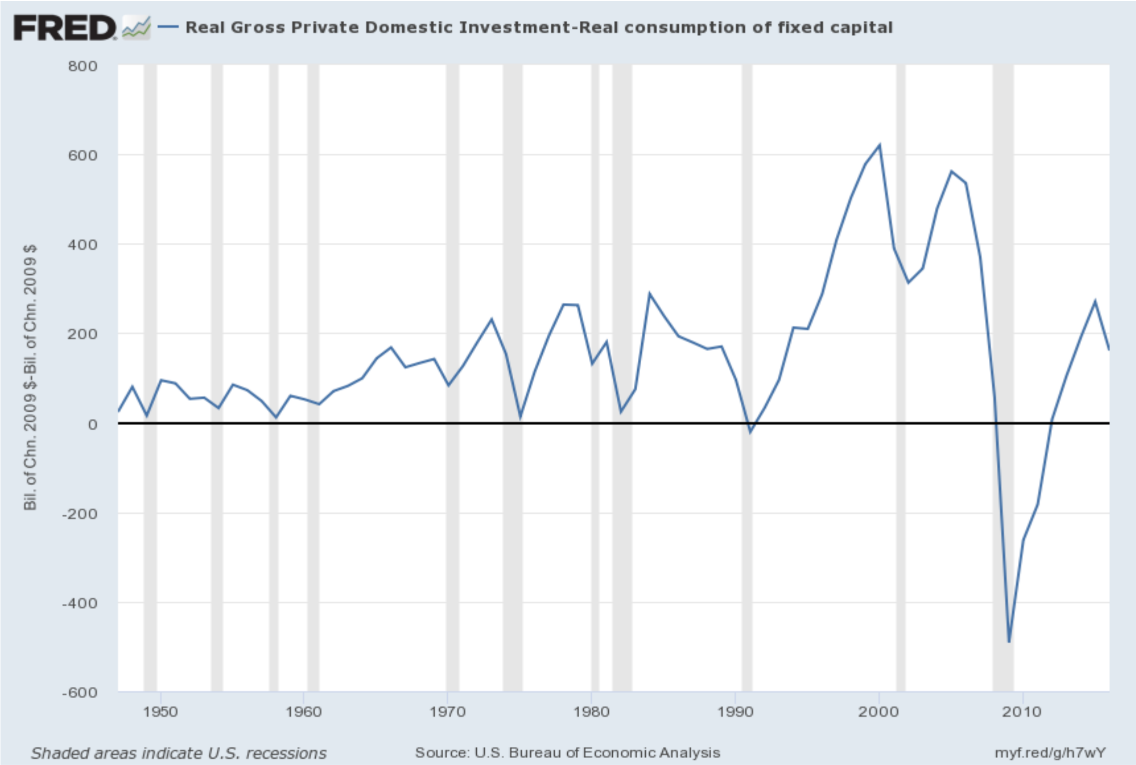
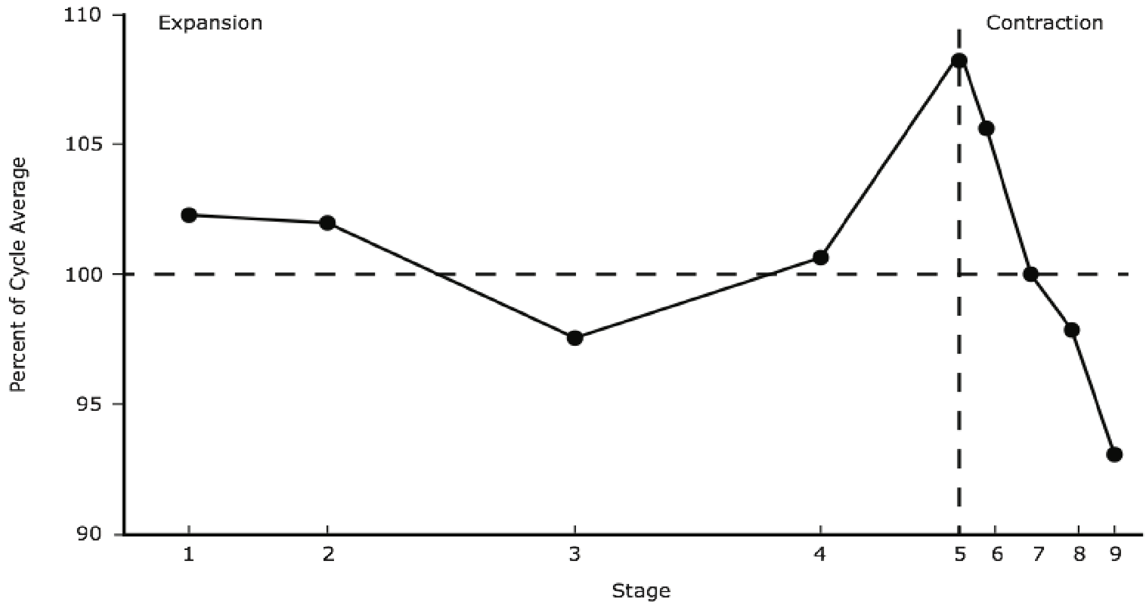


Figure 5. Ratio of raw material prices to consumer prices along the average U.S. cycle of 1970-2001



Source: Sherman (2010, p. 141). Note: Sherman divides the cycle in 9 stages. Stage 1 is the first quarter of the expansion. The rest of the expansion is divided into three pieces of equal length, stages 2, 3 and 4. Stage 5 is the expansion peak, just one quarter long. The recession is then divided into three equal pieces, stages 6, 7 and 8. Stage 9 is the last quarter of the recession.

Figure 6. Real interest rates over U.S. cycles, 1949-2017

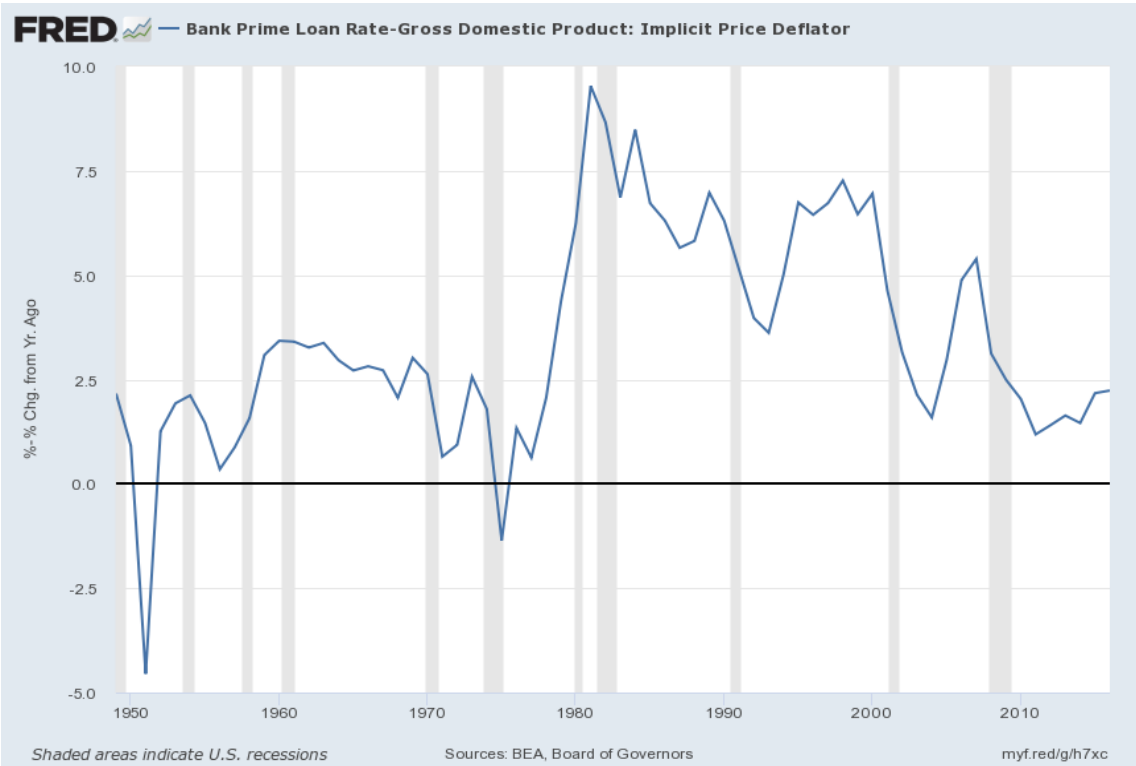


Figure 7. U.S. Government spending, 2007-2015

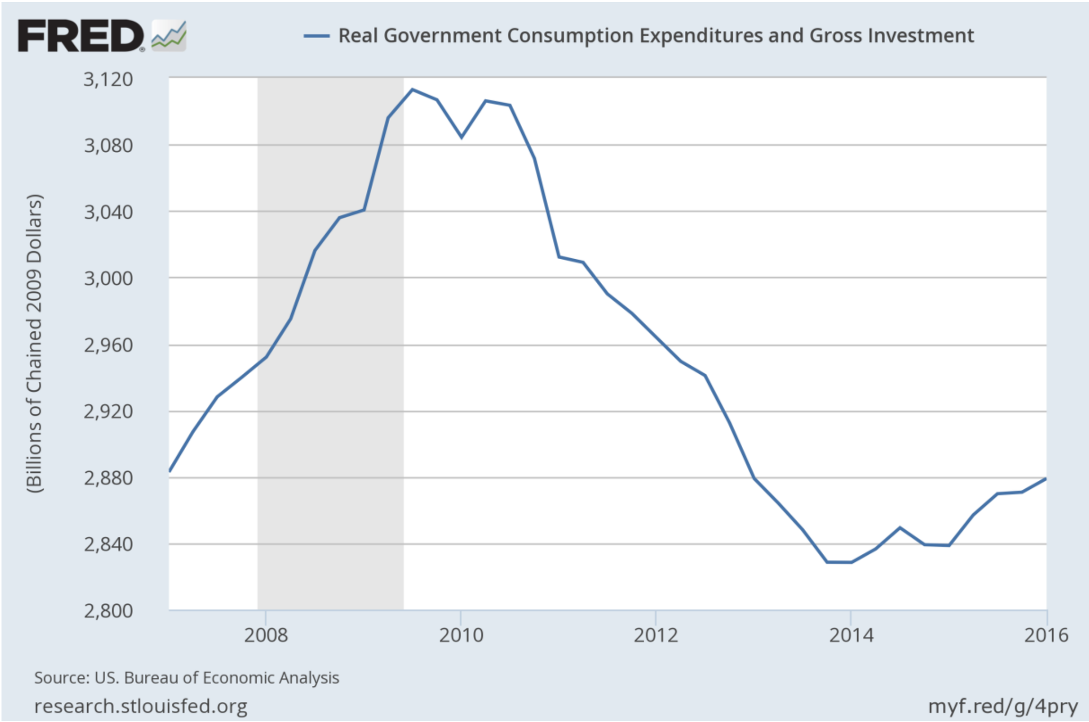
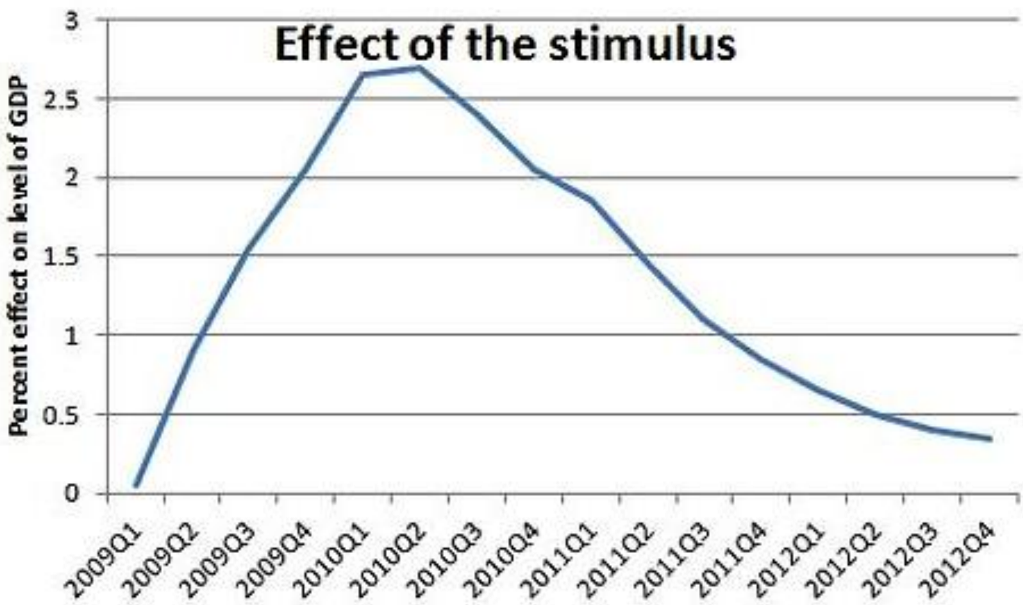


Figure 8. The effect on GDP of the Obama Stimulus as estimated by the U.S. Congress Budget Office



Source: Krugman (2011).

Figure 9. U.S. GDP, 2007-2015

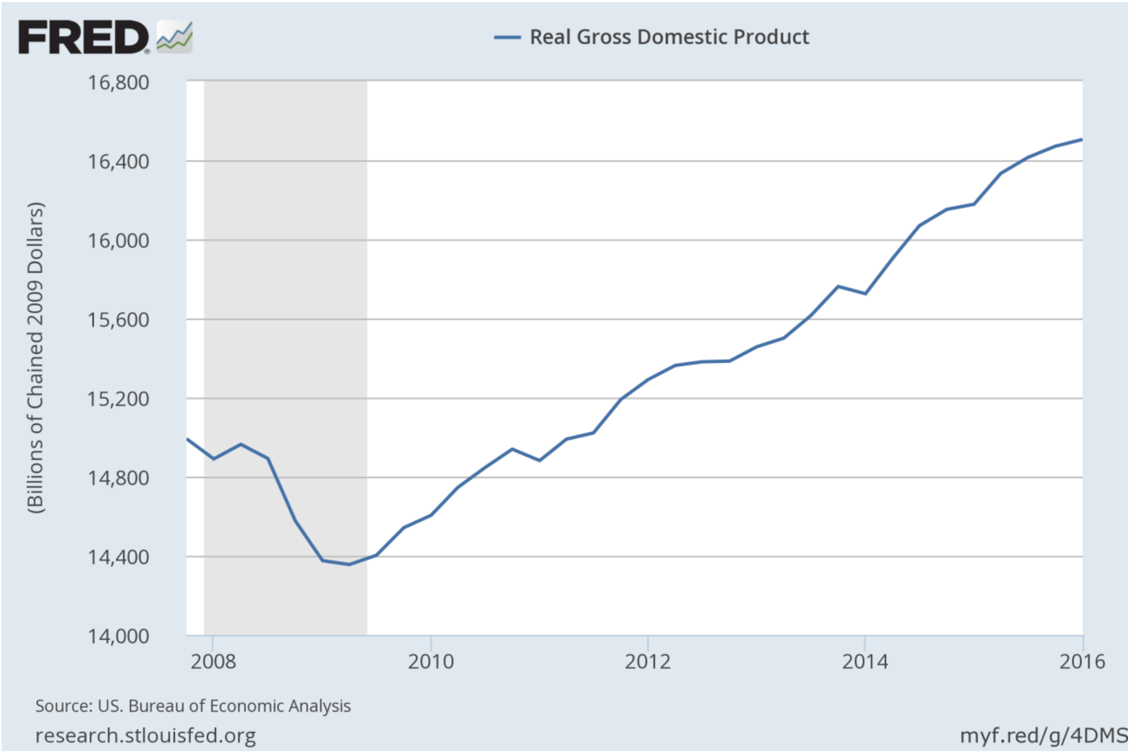


Figure 10. U.S. Capacity utilization, 2007-2015

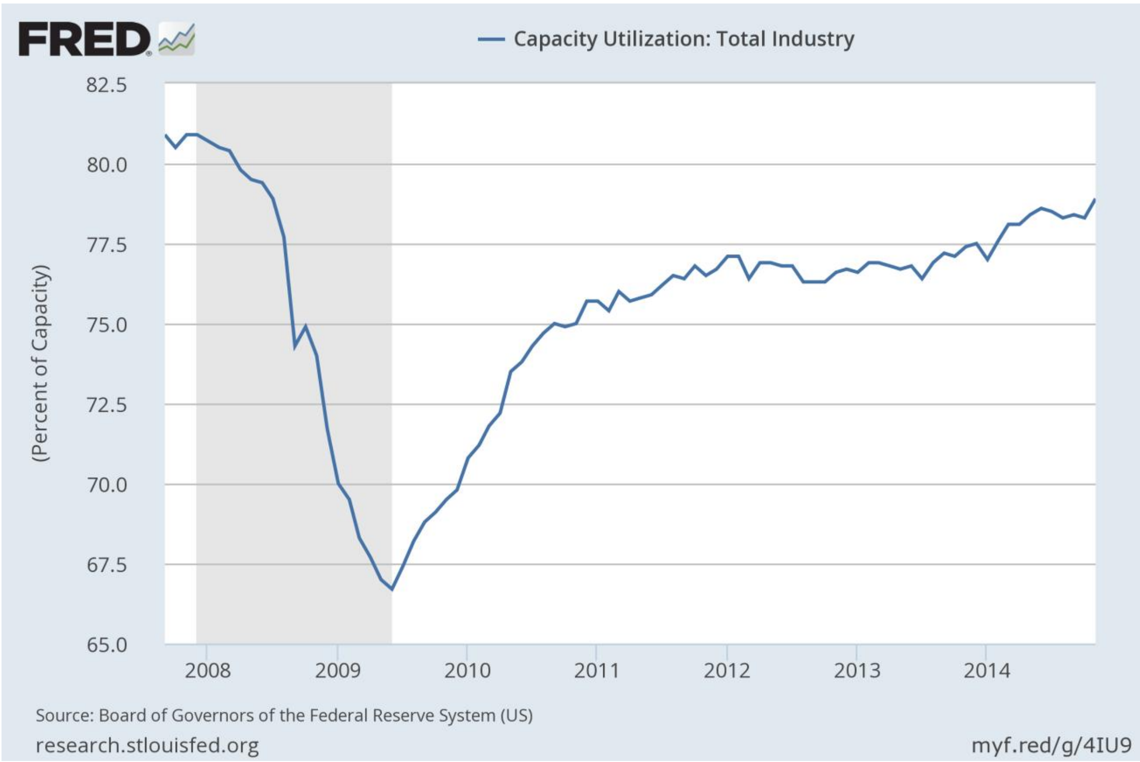


Figure 11. U.S. Corporate profits, 2007-2015

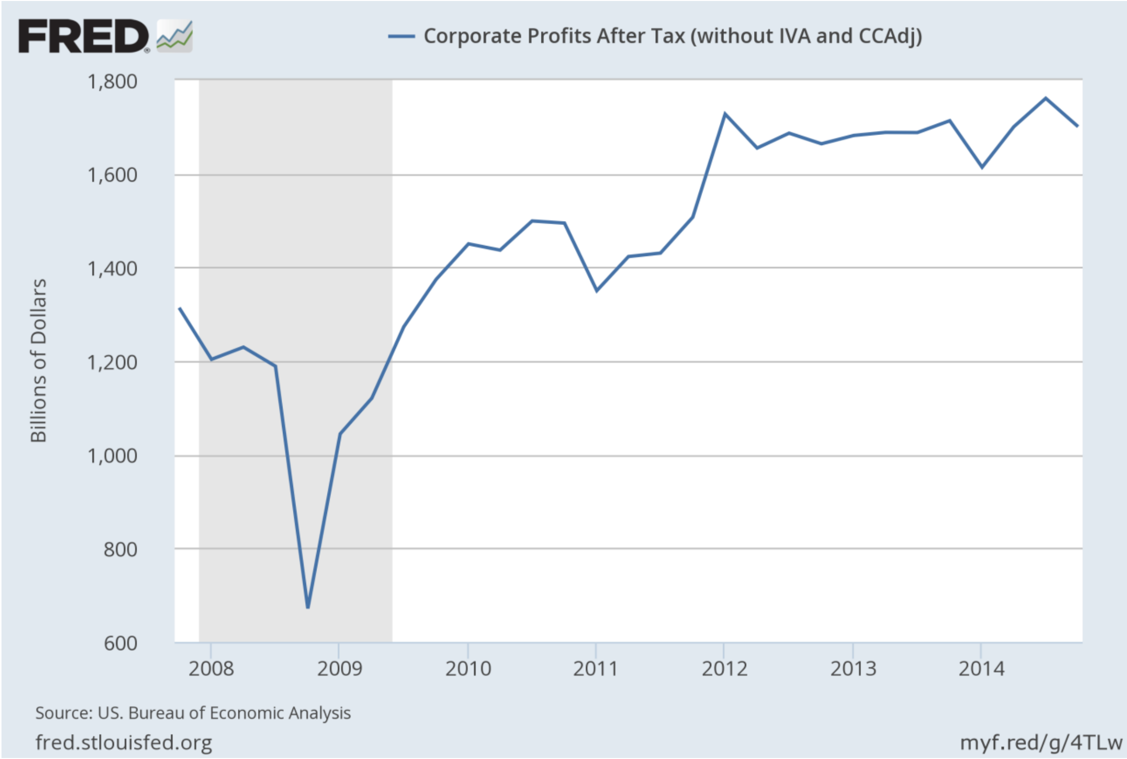


Figure 12. U.S. Private business investment, 2007-2015

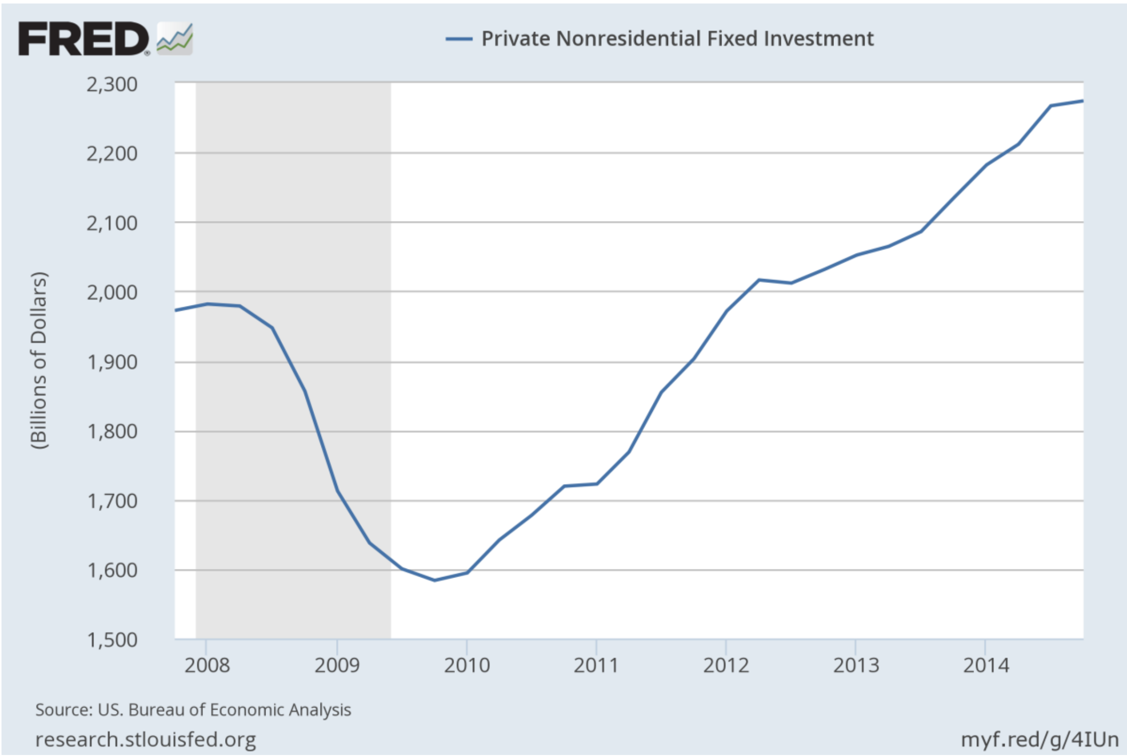


Figure 13. U.S. Government budget balance, 2007-2015

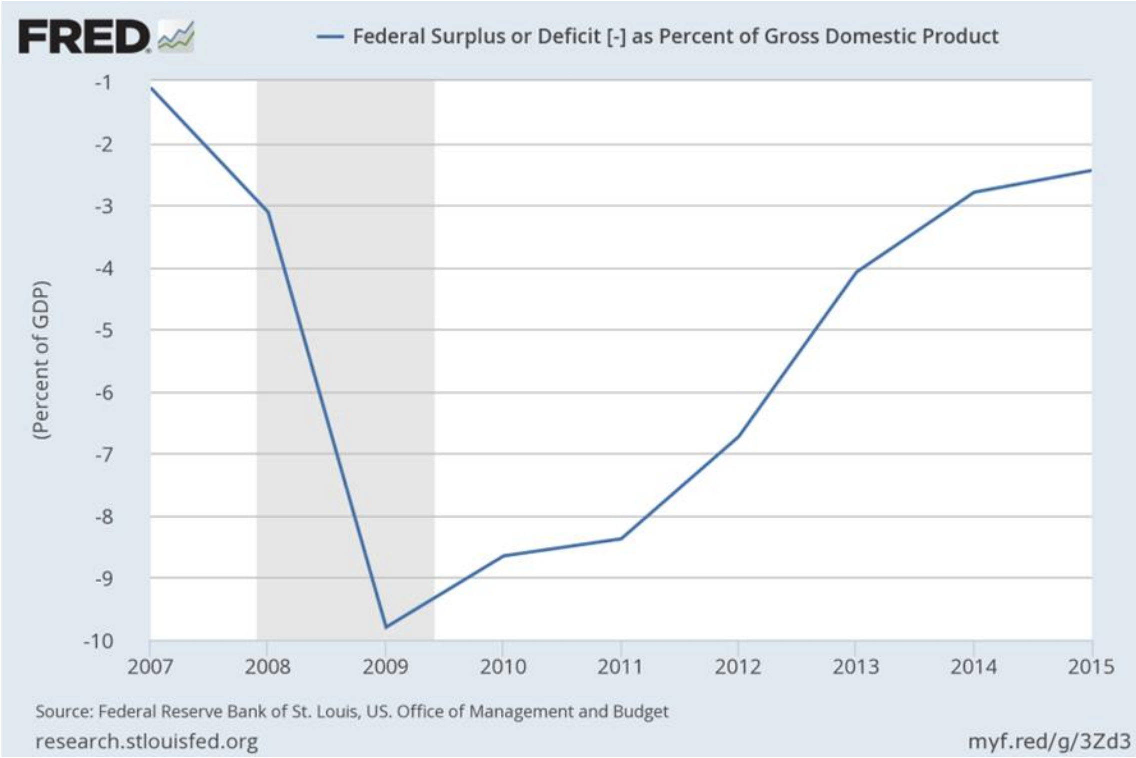


Figure 14. U.S. Public debt, 2008-2015

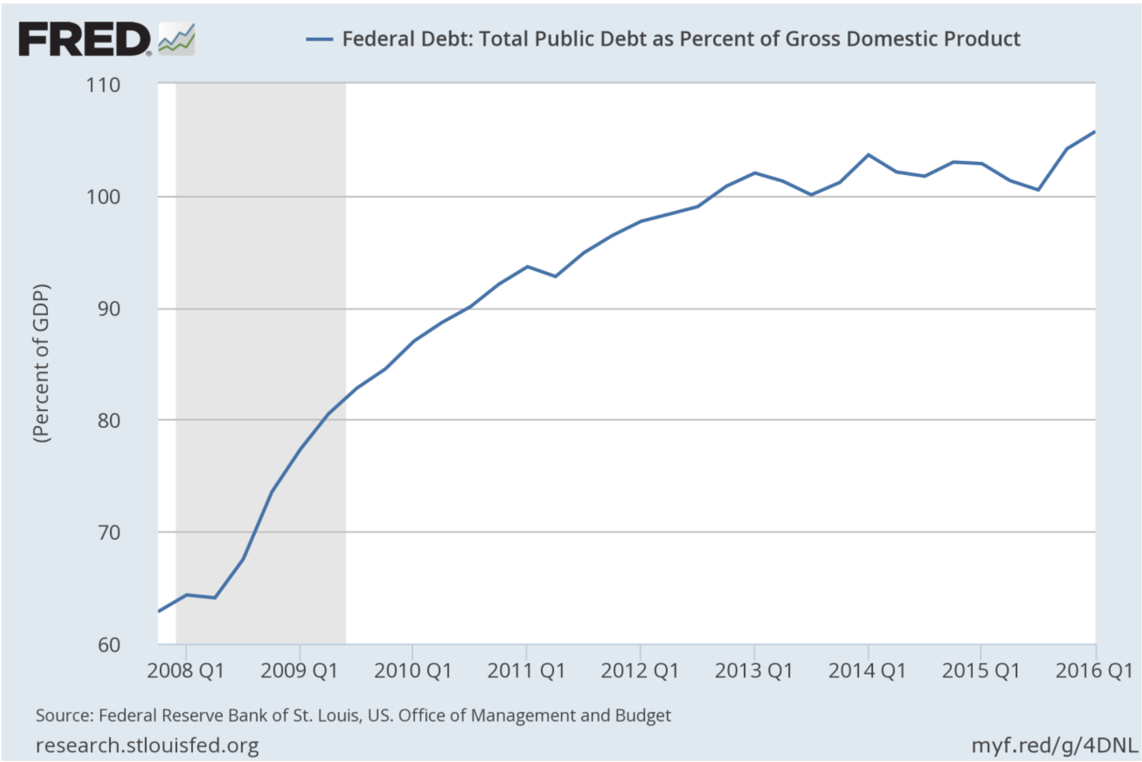


Table 1. Average changes of GDP, investment and profits in 10 U.S. cycles, 1950:1-2009:2

	Units	Expansions except the last year	Last year of expansions	Recessions
GDP	% change per year	5.63	3.50	-1.28
Investment	% change per year	15.93	6.57	-13.91
Profits	% change per quarter	12.5	-0.003	-5.0

Source: Harvey (2014, p. 397)

Table 2. Average changes in capacity utilization in seven U.S. cycles, 1967-2009:2

	Average change in percentage points	Average duration in months	Average change per month in p.p.
First half of expansions[a]	+5.72	31.2	+0.18
Second half of expansions[a]	+3.24	31.2	+0.10
Last part of expansions after the utilization peak	-1.59	9.6[b]	-0.17
Recessions	-8.04	14.6	-0.55

[a] The numbers are averages of only five of the seven cycles. The available data only starts in 1967 and thus prevents calculations for the expansion of the 1960s before the utilization peak. In turn, the expansion of the first cycle of the 1980s lasted only three quarters and prevents the same type of calculations.

[b] With the exception of the expansion of the 1990s, *utilization always peaked between six months and one year before the end of expansions.*

Source of the data: Federal Reserve Bank of St. Louis, series "Capacity Utilization: Total Industry", 1967-2009. Authors' calculations.

Table 3. Eurozone GDP and budget deficit, 2011-15

	Units	2011	2012	2013	2014	2015
GDP	Percent change	1.6	-0.9	-0.3	0.9	1.6
Budget deficit	Percent of GDP	4.2	3.7	3.0	2.6	2.0

Source: Ameco database

Table 4. Increases in investment and in demand in the various quarters of the expansion if the multiplier requires three quarters to exert its effect

Quarters	1^{st}	2^{nd}	3^{rd}	4^{th}	5^{th}
↑ Investment	\$10	\$10	\$10	\$10	\$10
↑ Consumption*	\$0	\$3	\$2+\$3	\$2+\$3	\$2+\$3
↑ Demand	\$10	\$13	\$15	\$15	\$15

* The first and the second rounds of consumption expenditure that take place in the first and the second quarters *after* the quarter of the increase in investment that generates them are assumed to be equal to \$3 and \$2, respectively.